Pure Appl. Geophys. 177 (2020), 27–36

https://doi.org/10.1007/s00024-019-02193-5

Pure and Applied Geophysics

Assessment of Space–Time Hazard of Large Aftershocks of the 2008 Mw7.9 Wenchuan Earthquake

Yuh-Ing Chen,[1] Jann-Yenq Liu,[2] and Hong-Wei Lai[1]

Abstract—To adaptively forecast forthcoming large aftershocks of the 2008 Mw7.9 Wenchuan, China, earthquake, we construct a new hazard model to describe the occurrence rate of aftershocks in a study region along the Longmenshan fault. The model is denoted by SRJ since it is a combination of the Reasenberg–Jones (RJ) model and a spatial hazard model that is obtained by taking a reference of previous earthquakes in the study region. We employ the maximum likelihood (ML) method to estimate the SRJ model based on aftershocks that occurred within 1 or 2 days after the Wenchuan earthquake. The probabilities of $M \geq 5.0$ aftershocks at each of the grids in the study region during next few days are then computed according to the estimated SRJ model, and hence the corresponding relative aftershock hazard (RAH) map is constructed. Finally, according to a variety of criteria for evaluating the hazard maps on depicting possible rupture area of forthcoming large aftershocks, the SRJ-based RAH map is demonstrated to be more efficient than the RAH maps constructed based on the RJ model incorporated with the gridding method using a fixed radius or varying radii.

Key words: Conditional distribution, lognormal normal distribution, maximum likelihood estimate, normal distribution, Reasenberg–Jones model, relative aftershock hazard map.

1. Introduction

Large earthquakes followed by a devastating main shock often bring a significant hazard to the area with a high-density population or weakened structures. To help decision makers to determine a low-risk emergent rescue work or an optimal treatment of damaged structures, the near real-time information about the hazard of large aftershocks is highly demanded. Therefore, an evaluation for the hazard of large aftershocks of a drastic main shock has been extensively studied. For example, the Gutenberg–Richter law (Gutenberg and Richter 1944) states the frequency–magnitude distribution of earthquakes. In addition, the Omori–Utsu law (Omori 1894; Utsu 1961; Utsu et al. 1995) describes the decaying occurrence rate of aftershocks at an elapsed time from the main shock. The Reasenberg–Jones (RJ) model (Reasenberg and Jones 1989, 1994), combining directly the Gutenberg–Richter and Omori–Utsu laws, then gives the time–magnitude hazard for an earthquake sequence after the main shock.

Basically, there are two important parameters in the RJ model. One is the b value in the Gutenberg–Richter law that measures the ratio of small to large earthquakes, and the other is the p value in the Omori–Utsu law that quantifies the power decay of aftershocks. In fact, both the b and p values are found to vary spatially (Kisslinger and Jones 1991; Smith 1981; Wiemer and Katsumata 1999; Wiemer and Wyss 1997). Hence, to depict a possible rupture area of future large aftershocks, a probabilistic aftershock hazard (PAH) map is constructed based on the RJ model incorporated with the gridding method (Wiemer and Wyss 2000). The PAH map is then used to forecast large aftershocks of the main shock in California (Gerstenberger et al. 2005, 2007).

In practice, the assessment of spatial hazard of aftershocks is of urgent need within a short time after the main shock in the study region. However, applying the gridding method with the maximum likelihood (ML) estimated RJ model (Aki 1965; Ogata 1983) may not be appropriate for near real-time forecast of aftershocks,

Electronic supplementary material The online version of this article (https://doi.org/10.1007/s00024-019-02193-5) contains supplementary material, which is available to authorized users.

[1] Institute of Statistics, National Central University, Taoyuan, Taiwan, Republic of China. E-mail: ychen5325@gmail.com
[2] Institute of Space Science, National Central University, Taoyuan, Taiwan, Republic of China.

especially when only sparse data are available in a short time after the main shock. For example, the gridding method with varying radii of equal data size may lose its locality for involving data overspread grids. On the other hand, the gridding method with a fixed radius is convenient to apply, but the grid may contain insufficient data to guarantee the reliability of model estimation. Therefore, to have a near real-time spatiotemporal forecast of aftershocks, we suggest constructing a space–time–magnitude hazard function for aftershocks. In this way, the model can be estimated based on available data. Probabilities of forthcoming large aftershocks can then be computed for all the grids in the study region based on the estimated model. Finally, the map of aftershock hazard can be obtained and used for locating the possible rupture area of such aftershocks.

Note that the great 2008 Mw7.9 Wenchuan shock with a focal depth of 19 km is a thrust and shallow earthquake and potentially brings up hazardous aftershocks (Zhang et al. 2008). Therefore, it is important to forecast large aftershocks in a potential rupture area (Fig. 1) within a short time after the main shock (Smyth et al. 2010). Since the epicenter of the Wenchuan earthquake at (31.00°N, 103.40°E) is near the Lonmenshan fault, it is reasonable to expect that aftershocks would majorly occur along the fault (Li et al. 2009).

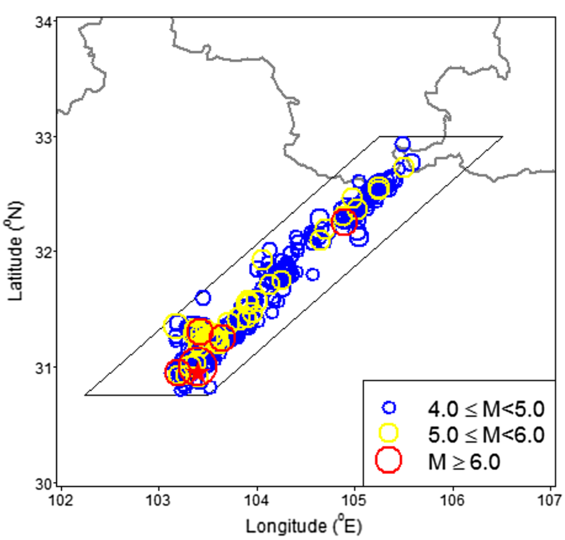

Figure 1
Aftershocks with magnitude 4.0 or larger that occurred during 2008/05/12–2008/05/19. The red star locates the epicenter of the 2008 May 12 Mw7.9 Wenchuan earthquake. The parallelogram cornered by (105.25°E, 33.00°N), (106.50°E, 33.00°N), (102.25°E, 31.00°N) and (103.40°E, 31.00°N) is the region under study

Hence, we consider a study region that covers the epicenter of the Wenchuan shock and the Longmenshan fault, and then construct a spatial hazard model for earthquakes in the region. Finally, we obtain a space–time–magnitude hazard model for aftershocks of the Wenchuan earthquake by combining the RJ model and the newly developed spatial hazard model. Therefore, this model is, hereafter, denoted by SRJ model.

The remainder of the paper is organized as follows. In Sect. 2, we introduce the SRJ model and show how to estimate the parameters by using the ML method. In Sect. 3, we report the results of an analysis of earthquakes after the Wenchuan shock and present the results of an evaluation of the related RAH maps. A discussion on how to use the proposed SRJ model for the hazard of aftershocks is given in Sect. 4. Finally, in Sect. 5, we draw conclusions for the research work with some remarks.

2. *Models and Methods*

2.1. *Time–Magnitude Aftershock Hazard Model*

The frequency–magnitude distribution of earthquakes according to the Gutenberg–Richter (Gutenberg and Richter 1944) law is

$$\log_{10} N(m) = a - bm,\ m > M_c, \qquad (1)$$

where $N(m)$ is the number of $M \geq m$ earthquakes, a is a constant related to the activity, b measures the ratio of small to large earthquakes and M_c is the cut-off magnitude for a complete earthquake catalogue. Note that we take $M_c = 4.0$ herein for completely recorded aftershocks of the Wenchuan earthquake (Wiemer and Wyss 2000). In fact, the magnitude of earthquakes is distributed according to a left-truncated exponential distribution. Therefore, the probability density function (pdf) of the earthquake magnitude is

$$h(m) = \beta \exp\{-\beta(m - M_c)\}, \quad m > M_c, \qquad (2)$$

and the probability of observing an $M > m$ earthquake is then given by

$$S(m) = \exp\{-\beta(m - M_c)\}, \quad m > M_c, \qquad (3)$$

where $\beta = b \ln 10$. Note that the maximum likelihood (ML) estimate of the parameter b and hence β can be obtained from Aki (1965).

The Omori–Utsu law (Omori 1894; Utsu 1961; Utsu et al. 1995) states the time-decaying occurrence rate of an aftershock sequence as

$$\lambda(t) = \frac{e^{\alpha}}{(t+c)^{p}},\ t > t_0, \tag{4}$$

where the constant e^{α} or α reflects the activity of aftershocks under study, c reveals the activity of aftershocks in the earliest stage, and the parameter p measures the decaying rate. Hence, the probability of aftershocks under study occurring in the time interval (T_1, T_2), $T_1 > t_0$, can be evaluated as

$$\begin{aligned} P(T_1, T_2) &= 1 - \exp\left\{-\int_{T_1}^{T_2} \lambda(t)\mathrm{d}t\right\} \\ &= 1 - \exp\{-e^{\alpha} A(T_1, T_2, c, p)\}, \end{aligned} \tag{5}$$

where $A(s,t,c,p) = \{(t+c)^{1-p} - (s+c)^{1-p}\}/(1-p)$ for $p \neq 1$ and $\ln(t+c) - \ln(s+c)$ for $p = 1$. Note that the ML estimates of the related parameters, α, c and p, can be obtained by maximizing the likelihood function in Ogata (1983). Therefore, the probability in (5) can be estimated by replacing the involved parameters with the corresponding ML estimates.

Assuming that the magnitude and occurrence time of aftershocks are independent, the RJ model (Reasenberg and Jones 1989, 1994) describes the occurrence rate of $M > m$ aftershocks as

$$\lambda(t, m) = \lambda(t)S(m),\ t > t_0 \text{ and } m > M_{\mathrm{c}}, \tag{6}$$

where $S(m)$ and $\lambda(t)$ are given in (3) and (4), respectively. Hence, the probability of at least one $M > m$ aftershock occurring during the time interval (T_1, T_2), $T_1 > t_0$ can be evaluated as

$$P(m, T_1, T_2) = 1 - \exp\{-e^{\alpha} A(T_1, T_2, c, p)S(m)\}. \tag{7}$$

Again, the ML estimates of the parameters in (6) can be obtained by maximizing the associate likelihood function, and hence the probability (7) can be estimated accordingly.

2.2. *Space–Time–Magnitude Aftershock Hazard Model*

We propose a bivariate distribution for possible epicenters of aftershocks of the Wenchuan earthquake where the Longmenshan fault is considered as a line source for the ruptures. The probability that an aftershock occurs at (x°E, y°N) in a study region then depends on how far it is from the epicenter of the main shock at (103.40°E, 31.00°N) and the fault. Note that 19 $M \geq 4.0$ earthquakes occurring before the main shock in the study region (Fig. 2) suggest a half-normal distribution with standard deviation $\sigma > 0$ for $z = \log(1 + |x - 103.4|)$. Hence, the pdf for z is obtained as

$$W(z) = \frac{2}{\sigma_z\sqrt{2\pi}} \exp\left\{-\frac{z^2}{2\sigma_z^2}\right\}, \quad z > 0. \tag{8}$$

Given that the earthquake occurred at x or z, the conditional probability that the earthquake occurs at (x°E, y°N) is then measured by how much it departs from the Longmenshan fault as represented by a linear function of x, namely, $\theta_0 + \theta_1 x$. The earthquakes that occurred before the main shock in the study region then suggest the conditional normal pdf of y given z with mean zero and $\sigma_y > 0$ as given by

$$W(y|z) = \frac{1}{\sigma_y\sqrt{2\pi}} \exp\left\{-\frac{(y - \theta_0 - \theta_1 x)^2}{2\sigma_y^2}\right\}, \quad -\infty < y < \infty. \tag{9}$$

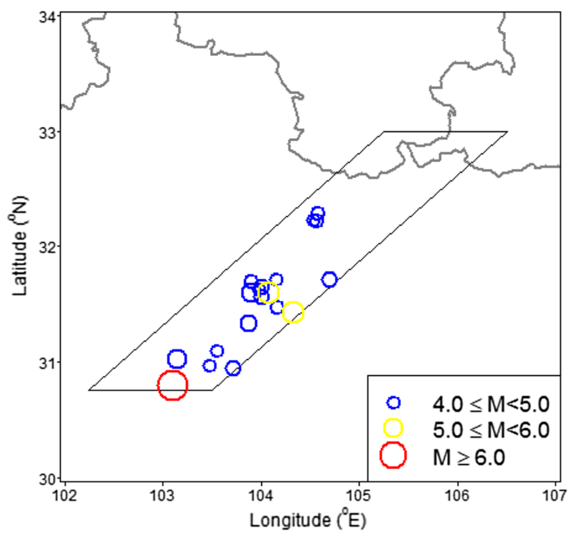

Figure 2
Earthquakes with magnitude 4.0 or larger that occurred during 1970/01/01–2008/05/11 in the parallelogram region under study

Note that, based on the 19 M $\geqq$ 4.0 earthquakes before the Wenchuan shock in the study region, the ML estimates of the parameters are $\hat{\theta}_0 = -53.99$, $\hat{\theta}_1 = 0.82$, $\hat{\sigma} = 0.52$ and $\hat{\sigma}_y = 0.21$. Moreover, the statistical goodness-of-fit test (Shapiro and Wilk 1965) gives observed significance levels as 0.701 and 0.107 for the normal distributions in (8) and (9), respectively. Therefore, we obtain the SRJ model that describes the occurrence rate of aftershocks of magnitude M at time t from the main shock with epicenter at (z, y) as

$$\lambda(t, m, z, y) = \lambda(t)h(m)w(z, y) \quad \text{for } t > t_0,\ m > M_c,\ -\infty < z, y < \infty, \tag{10}$$

and the associated pdf is

$$f(t, m, z, y) = g(t)h(m)w(z, y) \quad \text{for } t > t_0,\ m > M_c,\ -\infty < z, y < \infty, \tag{11}$$

where $w(z, y) = w(z)w(y|z)$, $h(m)$ is given in (2) and $g(t)$ is the pdf associated with $\lambda(t)$ in (4) as given by

$$g(t) = \frac{e^{\alpha}(p-1)c^{p-1}}{(t+c)^p}, \quad t > t_0.$$

Suppose that the study region can be partitioned into J grids as

$$R_j = \{(z, y) : z_{1j} \le z \le z_{2j}, y_{1j} \le y \le y_{2j}\},\quad j = 1, 2, \ldots, J$$

The probability of $M \ge m$ aftershocks occurring during the time interval (T_1, T_2), $T_1 > t_0$, in the grid R_j can be obtained as

$$P_j(m, T_1, T_2, R_j) = 1 - \exp\left\{-e^{\alpha}A(T_1, T_2, c, p)S(m)\iint_{R_j} w(z, y)\mathrm{d}z\mathrm{d}y\right\},\quad j = 1, 2, \ldots, J. \tag{12}$$

Let $D = \{(t_i, m_i, z_i, y_i),\quad i = 1, 2, \ldots, n\}$ be the available data set involving n aftershocks that are completely recorded within a short time after the main shock. Denote the parameter vector in the SRJ model by $\Theta = (\alpha,\ c,\ p,\ \beta,\ \theta_0,\ \theta_1)$. The ML estimates of parameters in (11) are then obtained by maximizing the likelihood function

$$L(\Theta|D) = \prod_{i=1}^{n} f(t_i, m_i, z_i, y_i) = \prod_{i=1}^{n} g(t_i)h(m_i)w(z_i, y_i). \tag{13}$$

Let $\hat{\Theta} = (\hat{\alpha},\ \hat{c},\ \hat{p},\ \hat{\beta},\ \hat{\theta}_0,\ \hat{\theta}_1)$ be the ML estimate of Θ. To evaluate the spatial hazard of $M \ge m$ aftershocks based on the SRJ model (10), we find the ML estimate of the probability (12) at each grid to obtain $\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_J$ at all the J grids in the study region. Since the probabilities are usually very small, we compute the relative hazards at each grid as $r_j = \hat{P}_j / \max\{\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_J\},\quad j = 1, 2, \ldots, J$ (Chen et al. 2015). Therefore, we obtain the relative aftershock hazard (RAH) map or the map of the relative hazards that can be used to depict the possible rupture area of forthcoming $M \ge m$ aftershocks.

3. Results

3.1. Relative Aftershock Hazard Maps

The ML estimated parameters in the SRJ model (11) summarized in Table 1 are computed based on Ms $\geq$ 4.0 aftershocks that occurred within 1 or 2 days after the Wenchuan earthquake in the study region (Fig. 1). Basically, the lines that describe the aftershock occurrence along the Longmenshan fault are the same for both the SRJ models estimated at day one and day two after the main shock. However, the two estimated RJ models are quite different, especially in the decaying rate of aftershocks. This is probably due to the fact that the number of $M \geq 4.0$ aftershocks at day one is more than twice than that at day two after the main shock. To see the difference, we present in Fig. 3 the fitted and forecasted occurrence rates of $M \geq 5.0$ aftershocks based on the estimated RJ models (6) when using data that occurred within 1 and 2 days after the main shock.

To investigate the spatial hazard for the forthcoming $M \geq 5.0$ aftershocks, the study region in the parallelogram (Fig. 1) is partitioned into J = 74 grids, where each grid is of size 0.2°E × 0.2°N. The SRJ model-based RAH map is constructed based on probabilities (12) of having $M \geq 5.0$ aftershocks

Table 1

Maximum likelihood estimates of parameters in the SRJ model based on $M \geq 4.0$ aftershocks that occurred within 1 or 2 days after the Wenchuan earthquake

Data	Parameter							
	α	c	p	β	θ_0	θ_1	σ_z	σ_y
1 day	4.18 (0.49)	0.30 (0.00)	0.76 (0.18)	2.10 (0.02)	− 47.36 (1.47)	0.76 (0.01)	0.48 (0.00)	0.12 (0.00)
2 days	7.44 (0.66)	0.86 (0.00)	1.58 (0.18)	2.19 (0.02)	− 48.06 (1.12)	0.77 (0.01)	0.53 (0.00)	0.11 (0.00)

The number in the parenthesis is the associated standard error

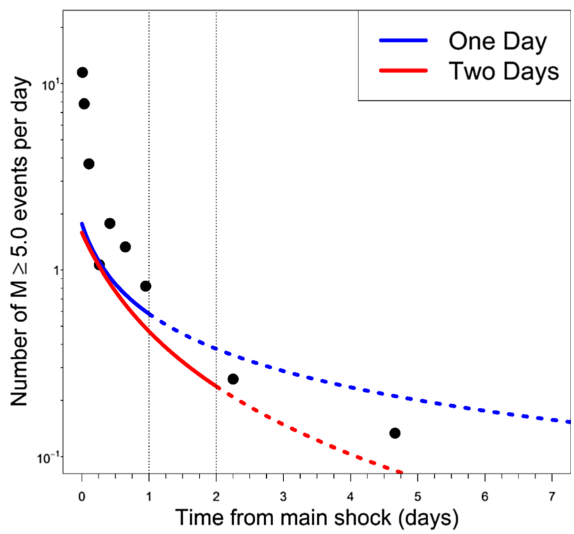

Figure 3
Occurrence rates of $M \geq 5.0$ aftershocks 7 days after the Wenchuan earthquake. Solid and dashed lines are the fitted and forecasted occurrence rates, respectively

during the next few days at each of the grids under study; hence, the relative hazards are calculated based on the estimated SRJ model. Note that the probabilities of having $M \geq 5.0$ aftershocks during next few days at each of the 74 grids can also be calculated based on the RJ model together with a gridding method (GRJ1 or GRJ2). To do so, we consider the GRJ1 method where each grid is centered with varying radii to include 20 $M \geq 4.0$ aftershocks. The RJ models at each of the 74 grids are estimated, the relative hazards are calculated and then the associate RAH map is obtained. Note that in the GRJ1-based RAH map, the relative hazards at any grids in the study region are of equal variation. We also consider the GRJ2 method by gridding with a fixed radius of 0.2° so that a sub-region center at each grid contains enough $M \geq 4.0$ aftershocks for a reliable RJ estimated model. In this paper, if the grids contain less than 20 observations, then the related probability and hence relative hazard is set to be zero. Therefore, grids with zero relative hazards are not counted in the study region of the RAH map. In fact, we constructed the RAH maps based on SRJ, GRJ1 and GRJ2 methods for alarming $M \geq 5.0$ aftershocks during the next 1–5 days at day one and day two after the Wenchuan earthquake. Note that the information of aftershock hazard is important within 3 days or 72 h after the great main shock. Hence, we only present herein two RAH maps in Figs. 4 and 5 for alarming $M \geq 5.0$ aftershocks at day one and day two, respectively, after the Wenchuan earthquake. In particular, Fig. 4 shows the hazard of aftershocks during the following 2 days, and Fig. 5 is for alarming aftershocks during the following day. The other RAH maps are then given as the supplement materials.

3.2. Evaluation of Relative Aftershock Hazard Maps

To assess how the RAH map works on alarming the forthcoming $M \geq 5.0$ aftershocks, we assign a positive sign (+) to the grid if it has a relative hazard greater than d for alarming the aftershocks, otherwise, the grid receives a negative sign (−) for non-alarming. We also find the number of grids with the aftershocks as denoted by E. Let TP(d) be the number of grids with + at which the aftershocks occurred and FP(d) the number of grids with + but without any such aftershocks. Then, the true positive rate and false positive rate are given as TPR(d) = TP(d)/E and FPR(d) = FP(d)/(74 − E), respectively. Hence, we obtain the receiver operating characteristic (ROC)

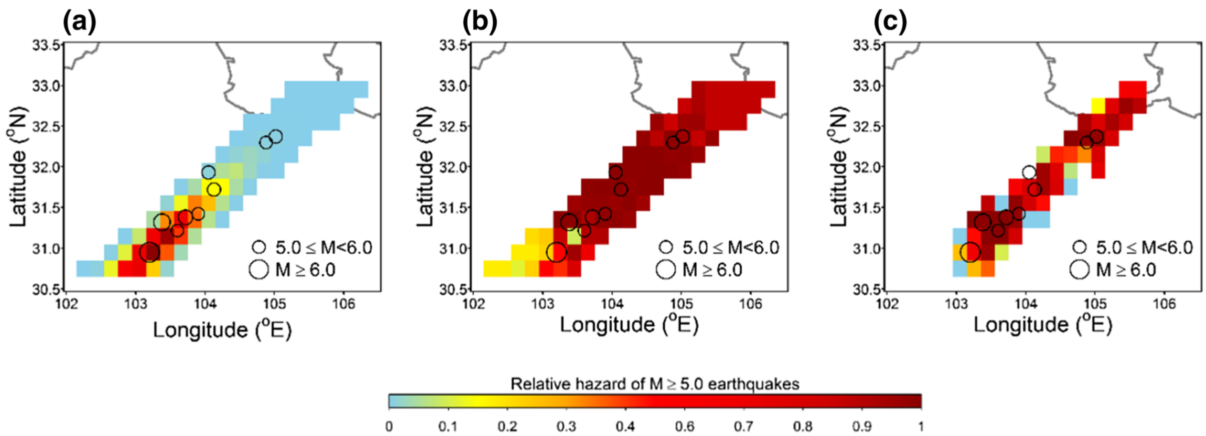

Figure 4
Relative hazard maps constructed based on $M \geq 4.0$ aftershocks occurring within 1 day after the Wenchuan earthquake for alarming $M \geq 5.0$ aftershocks during the next 2 days. **a–c** Are the maps constructed based on SRJ, GRJ1 and GRJ2 models, respectively. The black circles are the epicenters of the forthcoming aftershocks

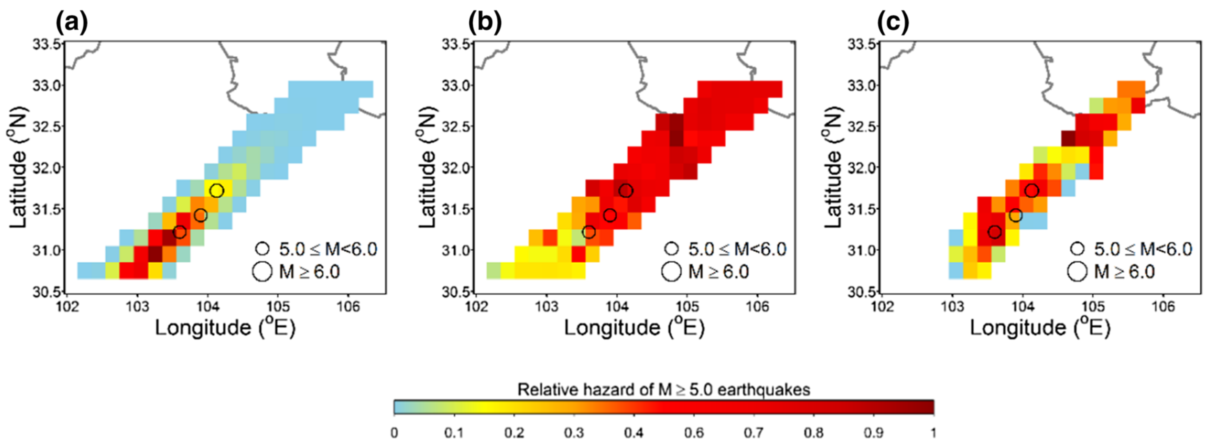

Figure 5
Relative hazard maps constructed based on $M \geq 4.0$ aftershocks occurred within 2 days after the Wenchuan earthquake for alarming $M \geq 5.0$ aftershocks during the next day. **a–c** Are the maps constructed based on SRJ, GRJ1 and GRJ2 models, respectively. The black circles are the epicenters of the forthcoming aftershocks

curve (Lusted 1960; Swets 1988) that is the curve of TPR(d) versus FPR(d) for all possible values of d. To see if the relative hazard is a valid marker for alarming the aftershocks, we compute the area under the ROC curve, denoted by AUC, for the RAH map which is, in fact, the Mann–Whitney statistics (Mann and Whitney 1947) comparing the relative hazards of the E and ($74 - E$) grids (Bamber 1975).

Note that the R score (Shi et al. 2001) is TPR(d) − FPR(d), but we find the largest R score to calibrate the optimal cut-off relative hazard, denoted by d^*. The resulted value of TPR(d^*) − FPR(d^*) is also known as Youden's index (Youden 1950). Based on the optimal cut-off value, we find the number of grids with positive signs as A, and then the positive predictive value (PPV) is TP(d^*)/A, which is the proportion of grids with the aftershocks among the alarming grids. Therefore, the probability gain of the RAH map-based alarming is obtained as PPV/PE, where PE = E/74 is the proportion of grids with the aftershocks among the grids under study. Since the probability gain may be sensitive to the PE and/or the number of alarming grids, we also compute the Bayes factor (Kass and Raftery 1995) that is the risk ratio or ratio of the odds of PPV and PE, namely, [PPV/(1 − PPV)]/[PE/(1 − PE)]. Finally, we present the four criteria in Table 2, including the AUC, Youden's index, probability gain and Bayes factor of each RAH map constructed based on SRJ, GRJ1 or GRJ2 for alarming the $M \geq 5.0$ aftershocks during the next 1–5 days at day one or two after the Wenchuan earthquake.

Note that AUC $\leq$ 0.5 implies that the relative hazard provides no help in alarming the forthcoming aftershocks. To test the hypothesis of AUC > 0.5 under significance level α, we then conduct the Mann–Whitney test (Mann and Whitney 1947) to see if the relative hazard in the cells with such aftershocks is stochastically larger than that in the cells without the aftershocks. Let U_i and V_j be the relative hazards of the cells in the study region with and without the aftershocks, respectively, $i = 1, \ldots, E$, $j = 1, \ldots, 74 - E$. The Mann–Whitney statistics is

$$Z = \frac{\sum_{i=1}^{E}\sum_{j=1}^{74-E} I\{U_i > V_j\} - E(74-E)/2}{\sqrt{E(74-E)(75)/12}}, \quad (14)$$

where $I\{a\} = 1$ if a is true, = 0, otherwise. The observed significance level is $1 - \Phi(z)$, where z is the observed Z value and $\Phi(\cdot)$ is the cumulative distribution of a standard normal distribution. Therefore, AUC > 0.5 is concluded if $1 - \Phi(z) \leq \alpha$. Moreover, since the Bayes factor is the ratio of the two odds, $\mathrm{TP}(d^*)/(A - \mathrm{TP}(d^*))$ and $E/(74 - E)$, we take the logarithm of the risk ratio as

$$W = \log\left(\frac{\mathrm{TP}(d*)}{A - \mathrm{TP}(d*)}\right) - \log\left(\frac{E}{74 - E}\right). \quad (15)$$

The associated standard error is

Table 2

Criteria of RAH map alarming for forthcoming $M \geq 5.0$ aftershocks within 1 or 2 days after the Wenchuan earthquake

Data	Criterion	Model	Forecasting period				
			1 day	2 days	3 days	4 days	5 days
1 day	AUC	SRJ	0.78[a]	0.82[a]	0.82[a]	0.82[a]	0.82[a]
		GRJ1	0.55	0.52	0.54	0.55	0.54
		GRJ2	0.54	0.63	0.62	0.61	0.61
	Youden index (cut point)	SRJ	0.54 (0.01)	0.64 (0.17)	0.64 (0.17)	0.64 (0.18)	0.64 (0.19)
		GRJ1	0.24 (0.91)	0.22 (0.97)	0.22 (0.93)	0.25 (0.97)	0.25 (0.99)
		GRJ2	0.22 (0.17)	0.33 (0.97)	0.33 (0.99)	0.33 (1.00)	0.33 (1.00)
	Probability gain	SRJ	2.00	3.60	3.60	3.60	3.60
		GRJ1	1.76	1.52	1.34	1.40	1.40
		GRJ2	1.24	2.52	2.52	2.52	2.52
	Bayes factor	SRJ	2.19	5.62[a]	5.62[a]	5.62[a]	5.62[a]
		GRJ1	1.89	1.64	1.40	1.49	1.49
		GRJ2	1.29	3.73	3.73	3.73	3.73
2 days	AUC	SRJ	0.91[a]	0.91[a]	0.92[a]	0.85[a]	0.72[a]
		GRJ1	0.23	0.24	0.19	0.34	0.44
		GRJ2	0.65	0.62	0.49	0.42	0.47
	Youden index (cut point)	SRJ	0.87 (0.30)	0.87 (0.31)	0.89 (0.32)	0.68 (0.33)	0.45 (0.33)
		GRJ1	0.01 (0.08)	0.01 (0.11)	0.01 (0.13)	0.14 (0.98)	0.21 (0.99)
		GRJ2	0.46 (0.32)	0.42 (0.44)	0.18 (0.25)	0.15 (0.27)	0.18 (0.57)
	Probability gain	SRJ	6.17	6.17	6.17	4.93	3.52
		GRJ1	1.01	1.01	1.01	2.96	3.02
		GRJ2	1.77	1.66	1.20	1.15	1.23
	Bayes factor	SRJ	7.89[a]	7.89[a]	8.75[a]	6.90[a]	4.79[a]
		GRJ1	1.01	1.01	1.01	3.45	3.83
		GRJ2	1.85	1.72	1.23	1.17	1.27

[a]Represents the criterion is significant at a level of 0.05

$$\mathrm{SE}(W) = \sqrt{\frac{1}{\mathrm{TP}(d*)} + \frac{1}{A - \mathrm{TP}(d*)} - \frac{1}{E} - \frac{1}{74 - E}}. \quad (16)$$

Hence, the standardized risk ratio is $Z = W/\mathrm{SE}(W)$. Similarly, if $1 - \Phi(z) \leq \alpha$, where z is the observed value of Z, then the Bayes factor is claimed to be greater than one (Morris and Gardner 1988). Hence, the criterion in Table 2 is marked if the data provides significant evidence for the hypothesis under a significance level of 0.05.

4. Discussion

The RJ models estimated based on $M \geq 4.0$ aftershocks that occurred within one day and two days after the Wenchuan earthquake (Fig. 3) tend to over- and under-forecast, respectively, the occurrence rate of future $M \geq 5.0$ aftershocks. This is not surprising because that the p value for 1-day data is much smaller than the one for 2-day data. Hence, the estimated RJ model at day one gives a larger occurrence rate of the following $M \geq 5.0$ aftershocks than does the RJ model estimated at day 2 after the main shock. Note that all the coefficients of variation (CV), namely, the ratios of standard errors and the related estimators, except the p value estimated based on 1-day data are less than 15%. As a matter of fact, the CV of 24% for the p value estimated based on 1-day data is still acceptable. Therefore, the two estimated RJ models are, in general, feasible for short-term forecast of the occurrence rate of future $M \geq 5.0$ aftershocks.

The aftershock hazard map constructed based on the gridding method together with the estimated RJ models is widely used to depict the possible rupture area for future large aftershocks in the study region (Wiemer and Wyss 2000; Gerstenberger et al. 2005, 2007). In each of Figs. 4 and 5, the RAH maps at panel (b) based on gridding with varying radii produce larger relative hazards than do the corresponding panel (c) when gridding with a fixed radius. This is because when data are sparse in the study region, the gridding method with varying radii for recruiting at least 20 $M \geq 4.0$ aftershocks for each grid would cross over many neighboring grids and hence lose its regional characteristics. At the same time, the associated estimated RJ model tends to over-forecast the occurrence rate of future $M \geq 5.0$ aftershocks. On the other hand, since the RJ model needs to be estimated with at least 20 $M \geq 4.0$ aftershocks at each grid, the RAH maps based on gridding with a fixed radius may not have relative hazards completely available at all the grids in the study region. Nevertheless, the AUC values (Table 2) suggest that both the gridding-based RAH maps are not only less competitive to the SRJ model-based RAH map, but also insignificant for efficiently depicting the possible rupture area for the forthcoming $M \geq 5.0$ aftershocks at day one or two after the Wenchuan earthquake.

The significantly large AUC values (Table 2) imply that the proposed SRJ model-based RAH maps, at panels (a) in Figs. 4 and 5, are valid for alarming $M \geq 5.0$ aftershocks within 1 or 2 days after the Wenchuan earthquake. By using the optimal cut-off relative hazards about 20% at day one and 30% at day two, all the SRJ model-based RAH maps preserve significantly large Bayes factors, except the case when alarming $M \geq 5.0$ aftershocks in the following day at day one after the main shock. The probability gains also show the consistent tendency. In fact, all the criteria suggest that when the SRJ model is estimated based on $M \geq 4.0$ aftershocks that occurred within 1 day after the main shock, the forecasting period should be 2–5 days for alarming the $M \geq 5.0$ aftershocks. When the SRJ model is estimated at day two after the main shock, however, it would be better to issue the alarming for $M \geq 5.0$ aftershocks during the next 1–3 days. These results are, in fact, reasonable since $M \geq 4.0$ aftershocks are heavily clustered within 1 day after the main shock, and then a longer period of forecasting of aftershock hazard is more plausible. On the other hand, the SRJ model estimated at day two after the main shock is preferred for a short-term forecast of a hazardous area of $M \geq 5.0$ aftershocks since the forecasted occurrence rate of such aftershocks may decay dramatically, as seen in Fig. 3.

5. Conclusion

Information about the short-term aftershock hazard is often urgently needed for ongoing rescue work, especially after a drastic main shock such as the great 2008 Mw7.9 Wenchuan, China, earthquake. In fact, a near real-time forecast for the hazardous area of the forthcoming large aftershocks is of particularly high demand. Previous researches show that the RJ model incorporated with the gridding method is successful to portray the hazard of aftershocks. However, when only sparse data are available in a short time after the main shock, the gridding method may not be able to reasonably reflect the regional aftershock hazard. Note that the epicenter of the Wenchuan earthquake is quite near the Longmenshan fault. Therefore, we physically consider the fault as a line source for the rupture of the forthcoming aftershocks, and proposed in this paper a space–time–magnitude model to describe the occurrence rate of the aftershocks. The proposed model is termed as the SRJ model since it is a combination of the RJ model and a spatial hazard distribution that was developed to provide a statistical goodness-of-fit to previous earthquakes in the study region covering the epicenter of the main shock and the Lonmenshan fault. The SRJ model can be estimated based on completely recorded aftershocks in a short time after the main shock, and hence the relative aftershock hazard (RAH) map can be constructed accordingly for portraying the relative hazard of forthcoming large aftershocks.

An analysis of the completely recorded aftershocks available at day one or two after the Wenchuan earthquake demonstrates that the RJ model is feasible for describing the time–magnitude hazard of large aftershocks. Four different criteria are then employed to evaluate the SRJ model-based RAH maps on depicting the possible rupture area of the forthcoming large aftershocks. The criterion of the area under the ROC curve confirms that the proposed RAH map is valid for alarming future large aftershocks. The criterion of Youden's index suggests an optimal cut-off relative hazard of about 20–30% for alarming the aftershocks. The probability gain and Bayes factor then conclude that the proposed RAH map along with the optimal cut-off relative hazard helps significantly depict the rupture area of the forthcoming large aftershocks. Therefore, the proposed SRJ model-based RAH map is of practical value for near real-time assessment of short-term aftershock hazard.

Acknowledgements

This research is supported by the Ministry of Sciences and Technology of Taiwan under the grant MOST 107-2119-M-008-018.

References

Aki, K. (1965). Maximum likelihood estimate of b in the formula logN = a-bM and its confidence limits. *Bulletin of Earthquake Research Institute Tokyo University, 43,* 237–239.

Bamber, D. C. (1975). The area above the ordinal dominance graph and the area below the receiver operating characteristic graph. *Journal of Mathematical Psychology, 12*(4), 387–415. https://doi.org/10.1016/0022-2496(75)90001-2.

Chen, Y. I., Huang, C. S., & Liu, J. Y. (2015). Statistical analysis of earthquakes after the 1999 Mw 7.7 Chi-Chi, Taiwan, earthquake based on a modified Reasenberg–Jones model. *Journal of Asia Earth Science, 114*(2), 299–304. https://doi.org/10.1016/j.jseaes.2015.02.018.

Gerstenberger, M., Jones, L. M., & Wiemer, S. (2007). Short-term aftershock probabilities: case studies in California. *Seismological Research Letters, 78*(1), 66–77. https://doi.org/10.1785/gssrl.78.1.66.

Gerstenberger, M., Wiemer, S., Jones, L. M., & Reasenber, P. A. (2005). Real-time forecasts of tomorrow's earthquakes in California. *Nature, 435*(7074), 328–331. https://doi.org/10.1038/nature03622.

Gutenberg, R., & Richter, C. F. (1944). Frequency of earthquakes in California. *Bulletin of the Seismological Society of America, 34*(4), 185–188.

Kass, R. E., & Raftery, A. E. (1995). Bayes factors. *Journal of the American Statistical Association, 90*(430), 773–795.

Kisslinger, C., & Jones, L. M. (1991). Properties of aftershocks in southern California. *Journal of Geophysical Research, 96*(b7), 11947–11958.

Li, Z. X., Chen, Z. L., Zhao, C. P., Shao, Z. G., Hua, W., Wang, Q., et al. (2009). Uneven aftershock distribution of Wenchuan Ms8.0 earthquake and possible mechanism. *Geodesy and Geodynamics, 2*(2), 7–12. https://doi.org/10.3724/SP.J.1246.2011.0007.

Lusted, L. B. (1960). Logical analysis in roentgen diagnosis. *Radiology, 74,* 178–193.

Mann, H. B., & Whitney, D. R. (1947). On a test of whwther one of 2 random variables is stochastically larger than the other. *Annals of Mathematical Statistics, 18*(1), 50–60.

Morris, J. A., & Gardner, M. J. (1988). Calculating confidence intervals for relative risks (odds ratios) and standardized ratios and rates. *British Medical Journal, 296*(6632), 1313–1316. https://doi.org/10.1136/bmj.296.6632.1313.

Ogata, Y. (1983). Estimation of the parameters in the modified Omori formula for aftershock sequences by the maximum likelihood procedure. *Journal of Physics of the Earth, 31*(2), 115–124. https://doi.org/10.4294/jpe1952.31.115.

Omori, F. (1894). On the aftershocks of earthquake. *Journal of the College of Science, Imperial University of Tokyo, 7,* 111–200.

Reasenberg, P. A., & Jones, L. M. (1989). Earthquake hazard after a mainshock in California. *Science, 243*(4895), 1173–1176. https://doi.org/10.1126/science.243.4895.1173.

Reasenberg, P. A., & Jones, L. M. (1994). Earthquake aftetshocks: Update. *Science, 265*(5176), 1251–1252.

Shapiro, S. S., & Wilk, M. B. (1965). An analysis of variance test for normality (complete samples). *Biometrika, 52*(3), 591–611. https://doi.org/10.1093/biomet/52.3-4.591.

Shi, Y., Liu, J., & Zhang, G. (2001). An evaluation of Chinese annual earthquake predictions. *Journal of Applied Probability, 38*(a), 222–231. https://doi.org/10.1239/jap/1085496604.

Smith, W. D. (1981). The b-value as an earthquake precursor. *Nature, 289*(5794), 136–139. https://doi.org/10.1038/289136a0.

Smyth, C., Mori, J., & Jiang, C. (2010). Model ensembles for prediction of Wenchuan aftershock activity. *Bulletin of Seismological Society of America, 100*(5B), 2532–2538. https://doi.org/10.1785/0120090300.

Swets, J. (1988). Measuring the accuracy of diagnostic systems. *Science, 240*(4857), 1285–1293. https://doi.org/10.1126/science.3287615.

Utsu, T. (1961). A statistical study on the occurrence of aftershocks. *Geophysical Magazine, 30*(4), 521–605.

Utsu, T., Ogata, Y., & Matsuura, R. S. (1995). The centenary of the Omori formula for a decay law of aftershock activity. *Journal of Physics of the Earth, 43*(1), 1–33. https://doi.org/10.2469/cfm.v24.n4.5.

Wieme, S. (2000). Introducing probabilistic aftershock hazard mapping. *Geophysical Research Letters, 27*(20), 3405–3408. https://doi.org/10.1785/0119990114.

Wiemer, S., & Katsumata, K. (1999). Spatial variability of seismicity parameters in aftershock zones. *Journal of Geophysical Research, 104*(b6), 13135–13151. https://doi.org/10.1029/1999JB900032.

Wiemer, S., & Wyss, M. (1997). Mapping the frequency-magnitude distribution in asperities; an improved technique to calculate recurrence times? *Journal of Geophysical Research, 102*(b7), 15115–15128. https://doi.org/10.1029/97JB00726.

Wiemer, S., & Wyss, M. (2000). Minimum magnitude of completeness in earthquake catalogs: Example from Alaska, the western US and Japan. *Bulletin of the Seismological Society of America, 90*(4), 859–869. https://doi.org/10.1785/0120020035.

Youden, W. J. (1950). Index for rating diagnostic tests. *Cancer, 3*(1), 32–35. https://doi.org/10.1002/1097-0142.

Zhang, Y., Feng, W. P., Xu, L. S., Zhou, C. H., & Chen, Y. T. (2008). Spatio-temporal rupture process of the 2008 great Wenchuan earthquake. *Science in China Series DL Earth Sciences, 52*(2), 145–154. https://doi.org/10.1007/s11430-008-0148-7.

(Received September 3, 2018, revised February 3, 2019, accepted April 9, 2019, Published online April 26, 2019)